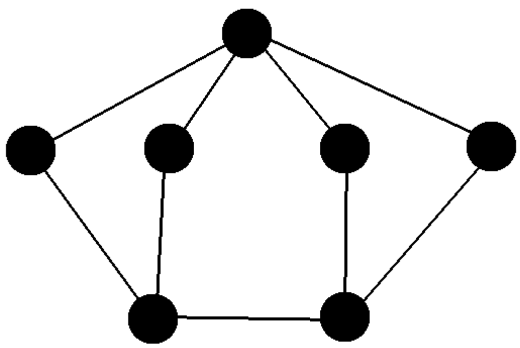
Graph Theory Final Practice Problems

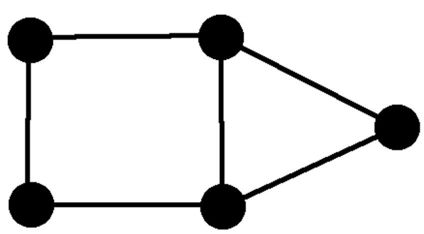
Use these as study aids in conjunction with the problems from the homeworks quizzes.

1. Draw the largest possible graph G (in terms of number of edges) with chromatic number of $\chi(G) = 4$ if $n(G) = 8$.

2. Given triangle-free graph G below with chromatic number $\chi(G) = 3$, draw a larger triangle-free graph G' such that $\chi(G') = 4$.



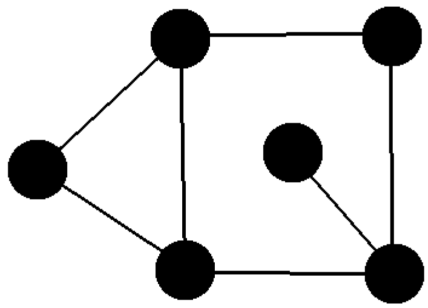
3. Given the below graph, use the edge contraction recurrence relation along with chromatic polynomials to determine how many possible ways we can properly color the vertices of the graph with k colors. Simplify your answer as much as possible.



4. Use a recurrence relation to compute how many possible spanning trees the above graph has.

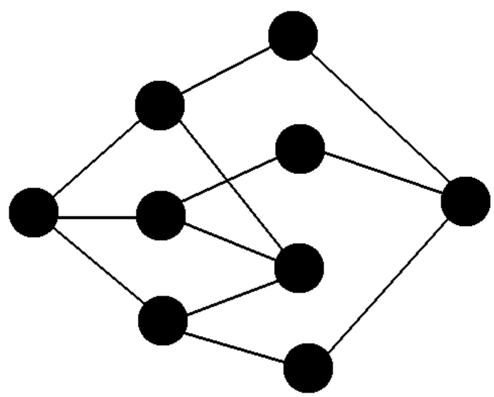
5. Draw a non-tree chordal graph of 5 vertices and give a simplicial elimination ordering of the graph. Is the graph you drew perfect? Why or why not?

6. Draw the dual graph of the planar graph G below. Label all faces of the given embedding of G and report their lengths. Show that G satisfies Euler's formula.



7. Connected graph G does not contain K_3 or $K_{3,3}$. It has 7 vertices and 9 edges. Explain why G must be planar or give a counter-example.

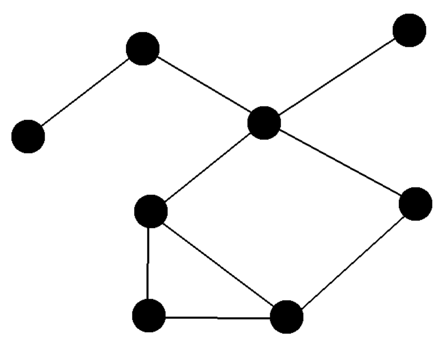
8. Draw a planar embedding of the graph given below or explain why no such planar embedding exists.



9. We have an algorithm to solve for an optimal vertex coloring which runs in exponential time. Suppose we also have an algorithm to solve an optimal edge coloring on an arbitrary graph in polynomial time. We wish to determine an optimal vertex coloring on G. Given that we know G contains no induced double odd triangle subgraphs or induced claws, how can we determine an optimal vertex coloring in polynomial time?

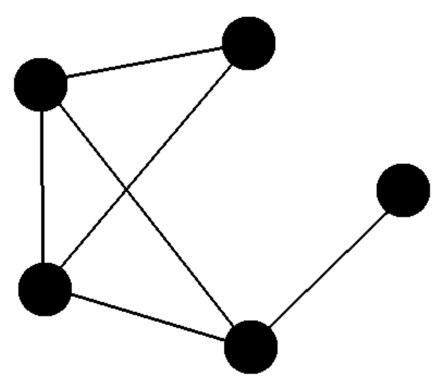
10. We have a connected graph G and its closure $H = C(G)$. The minimum value of $\lambda(u, v) = 5, \forall u, v \in V(H)$, where $\lambda(x, y)$ defines the number of internally disjoint paths between vertices x and y. H can be considered as a multi-partite graph with each partite set B_i being of maximal size, with the maximum size over all sets $max_i |B_i| = 4$. Is G Hamiltonian? Explain why or why not.

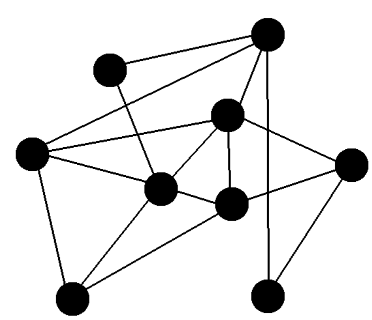
11. Draw the line graph $L(G)$ of graph G given below.



12. G is a biconnected bipartite graph with partite sets B_1 and B_2 where $|B_1| = |B_2|$. There exists no such $S \subseteq V(G)$ such that $c(G - S) > |S|$ where function $c(H)$ defines the number of components of a graph H. Is G Hamiltonian? Explain why it must be or give a counter-example.

13. Draw the closures $C(G), C(H)$ of the graphs G and H given below. Based on $C(G)$ and $C(H)$, how can we determine if each of G and H is Hamiltonian?





G.) H.)

14. Tree T has the following properties:

    (a) T has 112 vertices.

    (b) T is connected.

    (c) T is not color-critical but it is perfect.

    (d) Each non-leaf vertex $v \in V(T)$ has at least 3 neighbors but no greater than 6 neighbors.

    (e) The longest shortest u, v-path between any $u, v \in V(T)$ is 12.

    Give bounds on the chromatic number $\chi(T)$ of T such that the difference between the upper and lower bounds is as small as possible. Justify your bounds.

    Give bounds on the edge chromatic number $\chi'(T)$ of T such that the difference between the upper and lower bounds is as small as possible. Justify your bounds.

    What is the chromatic polynomial of the tree, $\chi(T, k)$? Simplify it as much as possible.

15. Prove that there always exists some optimal vertex ordering for the greedy coloring algorithm.